

A new approach with orthogonal array for global optimization in design of experiments

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Abstract In this paper, we develop an approach that determines the overall best parameter setting in design of experiments. The approach starts with successive orthogonal array experiments and ends with a full factorial experiment. The setup for the next orthogonal-array experiment is obtained from the previous one by either fixing a factor at a given level or by reducing the number of levels considered for all currently non-fixed factors. We illustrate this method using an industrial problem with seven parameters, each with three levels. In previous work, the full factorial of $3^7 = 2,187$ points was evaluated and the best point was found. With the new method, we found the same point using 3% of these evaluations. As a further comparison, we obtained the optimum using a traditional Taguchi approach, and found it corresponded to the 366th of the 2,187 possibilities when sorted by the objective function. We conclude the proposed approach would provide an accurate, fast, and economic tool for optimization using design of experiments.

Keywords Design of experiments · Orthogonal array · Factorial · Taguchi approach

1 Introduction

Introducing a new product or process design would give an effective way for a corporation to handle competition from its industry rivals. When a new design is introduced, several parameters (or factors) influence its objective characteristics. Each parameter could contain several optional levels for costs, safety, reliability, performance, etc. Thus, the parameters and their levels would form many optional settings, from which the ideal one is expected to be selected for the design.

Within the levels specified for factors of a design, most organizations would hope to determine the optimal or a workable setting through the design of experiment. Typically, two

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approaches are used in design of experiments, a full factorial experiment and a fractional factorial one.

A full factorial experiment conducts all possible combinations of the factor levels and therefore can reach the overall optimum setting, but it becomes overwhelming if the number of the design's parameters or levels increases. For example, if a new design involves 12 three-level parameters, the experiment needs to conduct $531441 (= 3^{12})$ settings. Therefore, the approach is only practical for a limited number of parameters and levels.

To overcome the full factorial experiment's limitation, a fractional factorial approach was developed originally by a British Statistician R. A. Fisher in the 1920s [1]. Fisher showed the full factorial array used in a full factorial experiment could be reduced to a smaller, but still statistically meaningful array, which is usually referred to as an orthogonal array.

An orthogonal array has two major requirements. The first is that the levels of any factor occur with the same frequency. The second is that, for any two factors, each possible *combination* of levels takes place with the same frequency. If all factors have q levels, an orthogonal array is usually expressed as $L_M(q^m)$ where m is the number of factors and M represents the number of rows in the array (a multiple of q^2).

In the study of orthogonal arrays, Taguchi [2] explored the entire design space with a few experiments and suggested several standard orthogonal arrays. He classified them into three types: 2-level arrays ($L_4, L_8, L_{16}, L_{32}, L_{64}$), 3-level arrays (L_9, L_{27}, L_{81}), and mixed 2-and-3-level arrays (L_{18}, L_{36}, L_{54}). For example, if one has 12 three-level parameters, Taguchi chooses the L_{27} design and so only conducts 27 specified settings. In this way, for a design containing many parameters and levels, a fractional factorial approach could determine a feasible parameter setting with much less effort and time because of using the orthogonal array.

Taguchi further defined a loss function to evaluate the deviation between an experimental value and a desired value. The loss function is a way to measure the performance characteristic deviation from the desired value. The value of a loss function was further redefined as a signal-to-noise (S/N) ratio. In general, a larger S/N ratio corresponds to a better design; that is, the design parameter setting is the one having the highest S/N ratio in the orthogonal array.

The Taguchi approach has been used widely in designing high-quality products at low cost in the engineering industry because the approach provides an acceptable solution effectively. Ghani et al. [3] applied the Taguchi approach in optimizing end milling parameters. Their results showed the optimal end milling parameters were determined with a minimum number of trials rather than a huge amount of trials in a full factorial design. Chen et al. [4] used finite element techniques and the Taguchi approach to optimize package design for plastic quad flat package (PQFP) with and without an assembled heat slug. Lin et al. [5] combined the grey relational analysis based on an orthogonal array and the fuzzy-based Taguchi approach to optimize the electrical discharge machining (EDM) process with multiple process responses.

However the design parameter setting determined by the Taguchi process is essentially a local optimum [6,7]. One must know the difference between this local optimum and a globally optimum in order to know the *quality* of the local optimum. Unfortunately, finding the global optimum can only be guaranteed via a full factorial experiment, which is prohibitive for many factors and levels. Consequently, limited research exists comparing the best two parameter settings from different approaches.

In this study a novel approach has been developed that creatively combines both the full and the fractional factorial approaches. The developed algorithm starts with several successive fractional factorial experiments and ends with a full factorial approach. In the algorithm we use the analysis of variance (ANOVA) to rank the involved factors according to their influences on the design's objective characteristics.

To verify the proposed approach, we applied it in designing a hatch corner having a groove with-a-hole for a 1092 TEU containership. The details in the application of the proposed approach will be described in the section. The results in this section show that our approach determines the globally best parameter setting using far less optional settings, <3% the number by a full factorial approach. The results also presented Taguchi approach's solution ranks far below the global optimum.

2 Algorithm of the proposed approach

The proposed algorithm starts with a series of fractional factorial experiments. The number of the series depends on the number of parameters of a new design. The parameters are gradually optimized for each fractional factorial step. Finally, a full factorial experiment is conducted for the remaining level-varying parameters to reach the globally best parameter setting. The algorithm was formulated below in 10 steps.

- Step 1: Define an initiated design and identify its objective characteristics.
- Step 2: Identify the factors (or parameters) affecting the design's objective characteristics.
- Step 3: Identify each factor's levels of the design.
- Step 4: Select a suitable orthogonal array by the number of factors to conduct a fractional factorial experiment.
- Step 5: Determine the design's objective value for each setting in the orthogonal array and weigh its influence by applying signal-and-noise (S/N) analysis.
- Step 6: Calculate the average S/N ratio of all the settings in the array, denoted as AV_{all} . Compute the average S/N ratio of the settings for each factor with a given level, denoted as AV_{lvl} . Plot the line graph by connecting each factor levels' AV_{lvl} to determine the predicted best parameter setting in this experiment (the point that combines the best level for each factor according to the main effects) Then determine actual S/N ratio value for the best setting by running the computer simulation; denote the result as S/N_{lg} .
- Step 7: Judge (a) if $S/N_{lg} > AV_{all}$, then go to the Step 8; or (b) eliminate the level with the worst AV_{lvl} value from each factor that has not yet been fixed to a single level and go to Step 4.
- Step 8: Determine the most influential factor in this experiment by analysis of variance and fix its level at the factor's best level. Since the determined most influential factor has its level fixed as the best one, we will call it a 'fixed factor' in the coming successive tests and the other factors the 'non-fixed' ones.
- Step 9: Examine whether the number of the non-fixed factors has been reduced enough to conduct a full factorial experiment. If yes, then go to Step 10; otherwise go to Step 4.
- Step 10: Conduct a full factorial experiment and then achieve the globally best parameter setting for the design.

It is worth mentioning if we end at Step 6 in the first fractional factorial experiment, our approach would obtain the same solution as Taguchi approach's.

3 Design optimization of a groove with-a-hole of a hatch corner

A hatch opening is needed for a ship to handle the access and engineering services. However, the opening causes discontinuities in ship structure. The discontinuities would result in stress

concentrations near the opening, decreasing ship’s safety navigating in an unpredictable sea state. Therefore, designing a hatch opening to improve stress concentrations used to be an essential task for shipbuilding engineers and researchers (Williams [8], Imamasa et al. [9], Huang [10]). Figure 1 is shown with three types of structure-strengthening designs typically used for a hatch opening.

The groove with-a-hole design in Fig. 1 starts at a point P (the point of tangency between the corner’s horizontal surface and the outer arc), penetrates through the corner’s vertical surface, and ends with a circular hole. An outer arc, a middle arc and an inner arc define the groove’s track. As a result, the design’s configuration will be determined by the arcs’ curvatures, the hole’s size and location, the p point’s location, which in the following were represented by six parameters defined in Fig. 2 as R_{out} , R_{md} , R_{in} , R_{hl} , θ and W_d . Besides, the shape penetrating on the corner’s vertical surface was considered another parameter influencing the stress distribution near the corner.

Zeng [11] numerically studied the hatch corner in a 1092 TEU containership by considering three different kinds of shapes shown in Fig. 3 and specifying the six parameters with

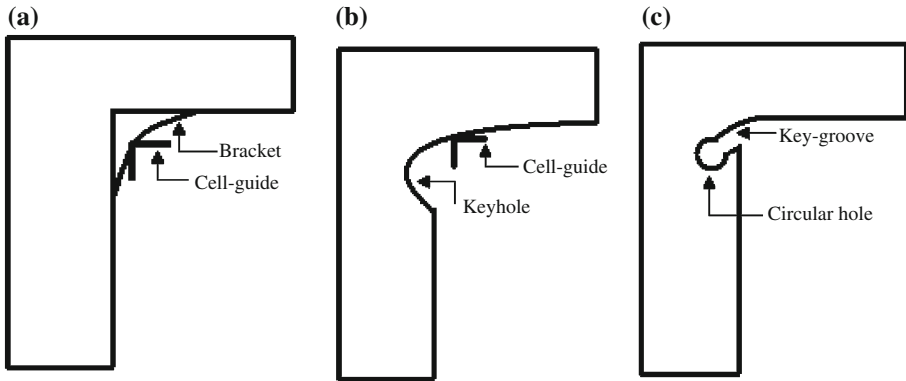


Fig. 1 Three different hatch corner devices with (a) a bracket, (b) a Keyhole, (c) a groove with-a-hole

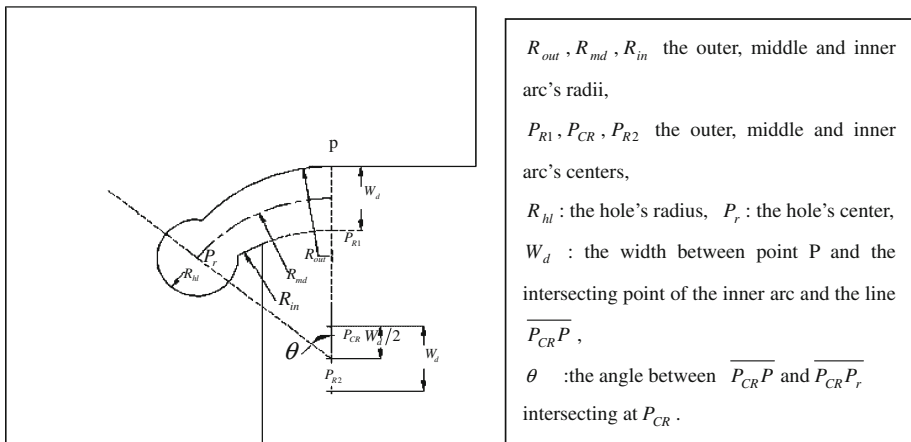


Fig. 2 The configuration of a groove with-a-hole at a hatch corner seen from the above

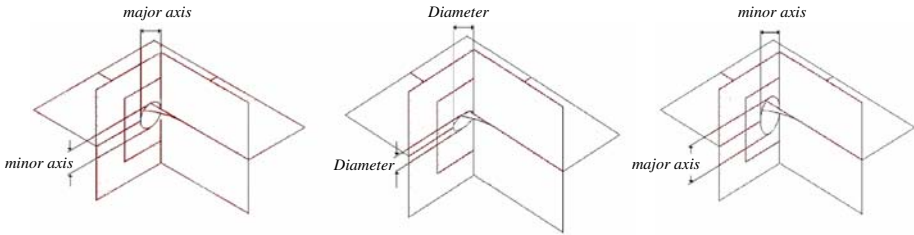


Fig. 3 Three different openings of a groove on the vertical surface of the corner

three levels each. He carried out 2,187 optional settings using a commercial structural code ANSYS [12] to determine the overall best one for the design.

The question of whether Zeng’s work be determined by the fractional factorial experiment sparked the present work. Therefore, the case by Zeng was used as a practical example to verify our approach developed in the study.

3.1 Description of parameters and levels

The 1092 TEU containership was designed by the United Ship Design and Development Center (<http://www.usddc.org.tw>). The designed speed is 18 knots, and the ship’s displacement is 20,180 ton. The ship’s general arrangement was shown in Fig. 4 and its main dimensions were displayed on Table 1. Soft and high-tension steels were used for ship construction. The equivalent yielding stress of the soft steel is 26 kgf/mm² and two different high-tension steels have equivalent yielding stresses of 32 kgf/mm² and 36 kgf/mm². The steel’s Young’s Modulus is 2.14 × 10⁴ kgf/mm² and the Poisson ratio is 0.3.

Zeng considered two elliptical shapes and one circular shape respectively to examine shape’s effect on the improvement in the stress concentration. In the following, the first ellipse was referred as to Type 1 having a ratio of its major axis in the vertical direction to its minor axis equal 2.0; the circular as Type 2; another ellipse as Type 3 with a ratio of its major axis in the horizontal direction to its minor axis equal to 1.5. Here, the length of the ellipse’s major axis links to the parameter W_d and the parameter R_{in} in a relationship of

$$\text{Major axis} = 0.5 \times \left\{ W_d + R_{in} - R_{in} \times \left[1 - (0.8/R_{in})^2 \right]^{1/2} \right\}$$

Hence, once W_d and R_{in} are given, the ellipse is determined. Note for the circular shape, the circle’s diameter equals the major axis.

The three various levels for the six parameters are presented in Table 2.

3.2 Description of numerical calculation

The hatch corner in the study sits at the section between Frame 40 and Frame 60 of the containership. The distributions of the mesh elements for the section and the fine grids near the groove with-a-hole in the work are shown in Fig. 5.

There were two stages in the numerical structural analysis of the ship. First, we used multi-celled ship’s structure stress analysis by a thin-walled member theory to get the rough numerical structural analysis of the entire ship subjected to wave loadings according to Lloyd’s Register, UK [13]. In the calculation we obtained the distributions of the twisting

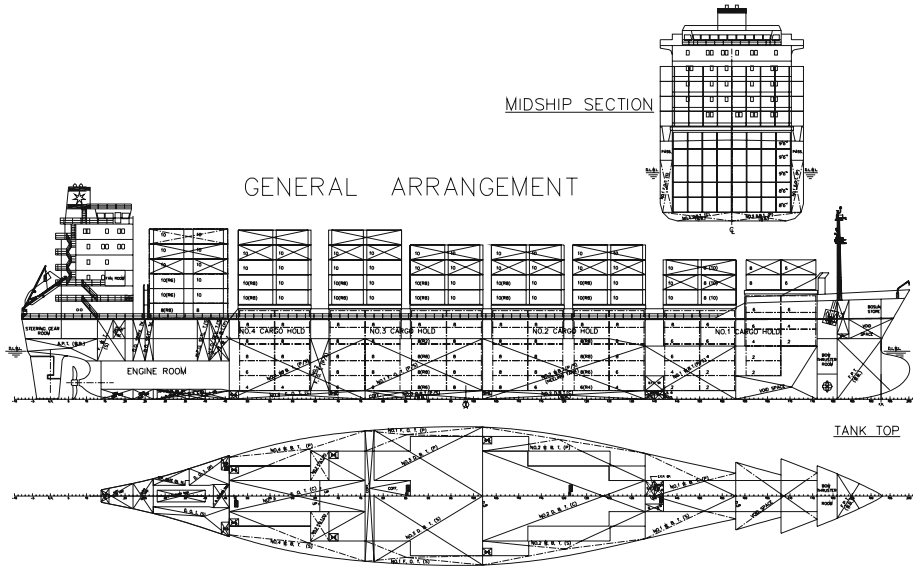


Fig. 4 The general arrangement of the 1092 TEU containership

Table 1 Main dimensions of 1092 TEU container ship

Length over all	155.0 m
Length between perpendiculars	145.0 m
Ship width	25.0 m
Ship depth	13.9 m
Designed draft	8.4 m

Table 2 The three various levels for the six parameters

Level	Parameter factor					
	W_d	θ	R_{md}	R_{hl}	R_{out}	R_{in}
1	0.3	55	1.4	0.30	1.3	1.3
2	0.4	60	1.5	0.35	1.5	1.5
3	0.5	65	1.6	0.40	1.7	1.7

angle, the deflection displacement, the shear center, the bending stress, the shear stress etc. in all sections. Second, some of the previous results were taken as the physical boundary conditions for the section of interest, and then the ANSYS code was used in this work to obtain the stress distributions over the hatch corner for this section.

In the following we used a symbol “ \aleph ” to represent the parameter, which has three levels of Type 1, Type 2, and Type 3. Therefore, this design involves seven three-level factors, and they are W_d , θ , R_{md} , R_{hl} , R_{out} , R_{in} and \aleph . Clearly, there would be $2187 (= 3^7)$ optional settings formed as if a solution pool, from which the ideal setting will be selected for the hatch corner.

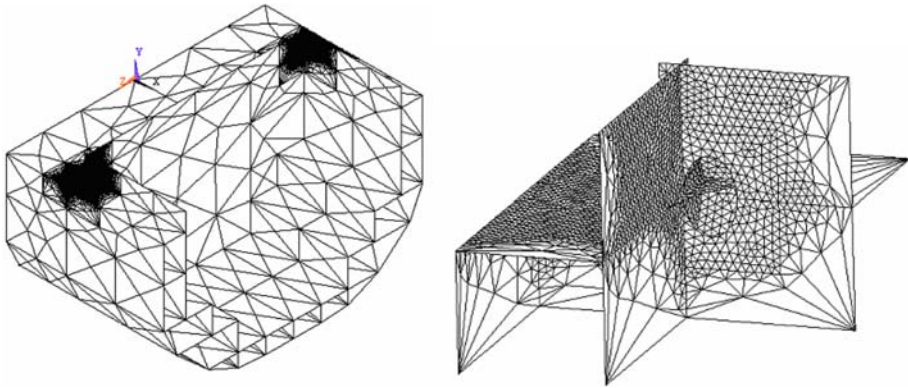


Fig. 5 The distributions of elements for the Frame40-60 Section and fine grids near the groove with-a-hole

The maximum equivalent stress near the groove defines the objective characteristics of the problem. Once a setting is picked from the pool, the stress is determined through ANSYS numerical analysis. To apply Taguchi’s signal/noise analysis, we related the value, denoted as y , to the S/N ratio in the form of

$$S/N = -10 \log y^2 \tag{1}$$

3.3 The first factorial experiment (fractional)

Since there are seven three-level factors for the case, we used a Taguchi’s orthogonal array L_{18} shown in Table 3. There are 18 settings arranged in the way required by the array. Next, processing the stress analysis for each setting, we determined the respective S/N ratio defined by Eq. 1. Accordingly, all 18 settings’ S/N ratio values were obtained and presented in the last column in Table 3.

As stated in the algorithm, AV_{lvl} represents the average S/N ratio for a factor at a given level. Its value represents the level’s weight on stress concentration distribution. The larger is the AV_{lvl} , the better is the design. After some calculations, we obtained 18 AV_{lvl} values for the seven factors, and the level’s influences for each factor were seen clearly by a line graph shown in Fig. 6. Therefore, in Fig. 6, the combination of each factor’s best level gave the best setting for the experiment, including level 1 of W_d , level 1 of θ , level 3 of R_{md} , level 3 of R_{hl} , level 2 of R_{out} and level 2 of R_{in} . For describing in the following text, we superscripted each factor by its best level number, and then expressed the best setting as $(W_d^1 = 0.3, \theta^1 = 55, R_{md}^3 = 1.6, R_{hl}^3 = 0.4, R_{out}^2 = 1.5, R_{in}^2 = 1.5, \aleph^3 = \text{Type 3})$.

Then, we ran the ANSYS code again for the best setting and then determined its S/N ratio as -24.4720 . Note the S/N ratio could be taken out of the array because the best setting could be one of the 18 settings tested. Next, we calculated the average S/N ratio of the 18 settings, AV_{all} , as -25.9160 . The fact the S/N_{lg} is greater than the AV_{all} satisfied the algorithm’s Step 7.

In the work the analysis of variance was used to determine the most influential of the seven factors in this experiment. To weigh a factor’s effect on the S/N ratio, we defined a parameter SSF for a factor as the sum of the difference squared between each AV_{lvl} and AV_{all} , and expressed it as

Table 3 The presentation of 18 settings and their S/N ratio values in the first factorial experiment

Trial no.	18 Seven-factor settings (L_{18} orthogonal array)							S/N ratio
	W_d	θ	R_{md}	R_{hl}	R_{out}	R_{in}	\aleph	
1	0.3	55	1.4	0.3	1.3	1.3	Type 1	-25.3828
2	0.3	60	1.5	0.35	1.5	1.5	Type 2	-24.6238
3	0.3	65	1.6	0.4	1.7	1.7	Type 3	-24.1661
4	0.4	55	1.4	0.35	1.5	1.7	Type 3	-24.9388
5	0.4	60	1.5	0.4	1.7	1.3	Type 1	-25.0948
6	0.4	65	1.6	0.3	1.3	1.5	Type 2	-25.1670
7	0.5	55	1.5	0.3	1.7	1.5	Type 3	-24.7780
8	0.5	60	1.6	0.35	1.3	1.7	Type 1	-24.9275
9	0.5	65	1.4	0.4	1.5	1.3	Type 2	-24.8593
10	0.3	55	1.6	0.4	1.5	1.5	Type 1	-24.0759
11	0.3	60	1.4	0.3	1.7	1.7	Type 2	-25.6642
12	0.3	65	1.5	0.35	1.3	1.3	Type 3	-24.5114
13	0.4	55	1.5	0.4	1.3	1.7	Type 2	-24.5649
14	0.4	60	1.6	0.3	1.5	1.3	Type 3	-24.9967
15	0.4	65	1.4	0.35	1.7	1.5	Type 1	-25.4850
16	0.5	55	1.6	0.35	1.7	1.3	Type 2	-24.8269
17	0.5	60	1.4	0.4	1.3	1.5	Type 3	-24.7443
18	0.5	65	1.5	0.3	1.5	1.7	Type 1	-25.6796

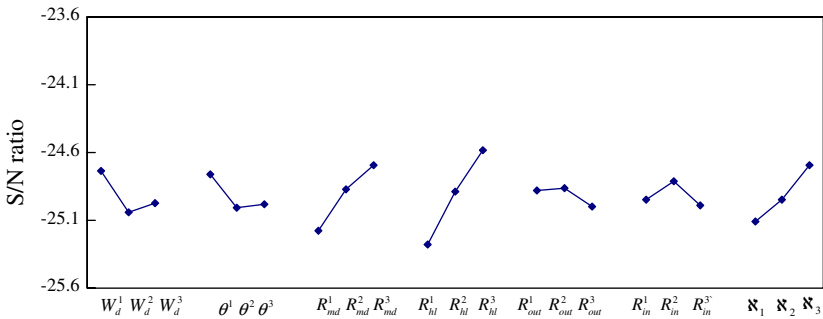


Fig. 6 The line graph of lines formed by three-level AV_{lvl} versus seven respective factors

$$SSF = \sum_{i=1}^{nl} ns(AV_{lvl}^i - AV_{all})^2 \tag{2}$$

where nl represents the number of a factor’s levels and ns stands for the number of settings with a given level. In this analysis, $ns = 6$ and $nl = 3$. Therefore, the SSF of the W_d factor was calculated by

Table 4 SSF values of the tested factors for the five respective fractional factorial experiments

Fractional factorial experiment	SSF of the tested factors						
	W_d	θ	R_{md}	R_{hl}	R_{out}	R_{in}	\aleph
First	0.1512	0.1092	0.3613	0.7262	0.0344	0.0514	0.2681
Second	0.0773	0.0088	0.6466		0.0399	0.0964	0.2058
Third	0.0728	0.0114			0.0607	0.0451	0.3277
Fourth	Orthogonal array changes from L_{18} into L_8						
Fifth	0.0804	0.0020			0.0006	0.0015	

$$\begin{aligned}
 \text{SSF} &= \sum_{i=1}^3 ns(AV_{lvl}^i - AV_{all})^2 = 6(-24.7374 + 24.9156)^2 + 6(-25.0412 + 24.9156)^2 \\
 &\quad + 6(-24.9693 + 24.9156)^2 = 0.1512
 \end{aligned}$$

So, the other six factors’ SSF values were calculated and presented in Table 4. Table 4 clearly shows the factor R_{hl} in this experiment as the most influential in improving stress concentration. Also, the level 3 (0.4) was selected as the factor’s most influential one among the three levels.

3.4 The second factorial experiment (fractional)

The orthogonal array L_{18} was used again in the second experiment for six non-fixed factors and one fixed factor, $R_{hl} = 0.4$. Also, 18 settings were arranged in the array shown in Table 5. Since six of the 18 settings already existed in Table 3, only the other 12 settings needed to be conducted by ANSYS code. The 18 settings’ S/N ratios are presented in Table 5.

The line graph for the six factors was shown in Fig. 7. Compared with the results in Fig. 6, we found all AV_{lvl} values in Fig. 7 are larger than -25.0 . Besides, for each factor the deviation between its greatest and lowest AV_{lvl} values became narrower than the one in the last experiment. The fact justified the factor R_{hl} as the most influential among the seven. Figure 7 shows the best setting as ($W_d^1=0.3, \theta^2=60, R_{md}^3=1.6, R_{hl}=0.4, R_{out}^3=1.7, R_{in}^2=1.5, \aleph^2=Type\ 2$). After running ANSYS code for the new setting, we determined its S/N ratio value, S/N_{lg} , as -24.3644 . From Table 5, we calculated the average S/N ratio of the 18 settings, AV_{all} , as -24.6193 .

The six factors’ SSF values were calculated and displayed in Table 4, from which the factor R_{md} was seen as the most influential in this experiment because of having the largest SSF value. Figure 7 shows the level 3 was found as the factor’s best level.

3.5 The third factorial experiment (fractional)

We kept using the orthogonal array L_{18} in the third experiment for five non-fixed factors and two fixed factors, $R_{hl}=0.4, R_{md}=1.6$. The 18 different settings in the array are shown in Table 6. Since six of the 18 settings were found in Table 5, like the second experiment, this experiment was completed with 12 ANSYS numerical analyses for as many new settings.

Table 5 The presentation of 18 settings and their S/N ratio values in the second factorial experiment

Trial no.	18 Seven-factor settings (L_{18} orthogonal array)							S/N ratio
	W_d	θ	R_{md}	R_{hl}	R_{out}	R_{in}	\aleph	
1	0.3	55	1.4	0.4	1.3	1.3	Type 1	-25.3152
2	0.3	60	1.5	0.4	1.5	1.5	Type 2	-24.3865
3	0.3	65	1.6	0.4	1.7	1.7	Type 3	-24.1661
4	0.4	55	1.4	0.4	1.5	1.7	Type 3	-25.2300
5	0.4	60	1.5	0.4	1.7	1.3	Type 1	-25.0948
6	0.4	65	1.6	0.4	1.3	1.5	Type 2	-24.3665
7	0.5	55	1.5	0.4	1.7	1.5	Type 3	-24.5639
8	0.5	60	1.6	0.4	1.3	1.7	Type 1	-24.5880
9	0.5	65	1.4	0.4	1.5	1.3	Type 2	-24.8593
10	0.3	55	1.6	0.4	1.5	1.5	Type 1	-24.0759
11	0.3	60	1.4	0.4	1.7	1.7	Type 2	-24.4787
12	0.3	65	1.5	0.4	1.3	1.3	Type 3	-24.5072
13	0.4	55	1.5	0.4	1.3	1.7	Type 2	-24.5649
14	0.4	60	1.6	0.4	1.5	1.3	Type 3	-24.1688
15	0.4	65	1.4	0.4	1.7	1.5	Type 1	-24.7025
16	0.5	55	1.6	0.4	1.7	1.3	Type 2	-24.1554
17	0.5	60	1.4	0.4	1.3	1.5	Type 3	-24.7443
18	0.5	65	1.5	0.4	1.5	1.7	Type 1	-25.1795

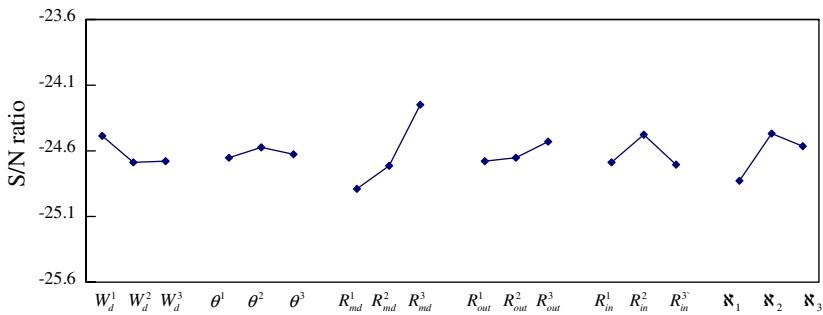


Fig. 7 The line graph of lines formed by three-level $AV_{I_{vl}}$ versus six respective factors

Then, the 18 settings’ S/N ratio values were determined and presented in the last column in Table 6.

The S/N ratio values of the 18 settings were found closer in this experiment than the previous two experiments. The results of the line graph in Fig. 8 revealed the best setting in this experiment as ($W_{d^3} = 0.5, \theta^1 = 55, R_{md} = 1.6, R_{hl} = 0.4, R_{out}^3 = 1.7, R_{in}^2 = 1.5, \aleph^3 =$ Tape 3). After completing the ANSYS analysis for the chosen setting, we determined its S/N ratio value, S/N_{I_g} , as -23.9263 . Next, the average S/N ratio of the 18 settings, AV_{all} , was calculated from Table 7 as -24.3765 .

Table 4 shows the factor \aleph as the most influential one in this experiment, and Fig. 8 shows Type 3 as the factor’s best level.

Table 6 The presentation of 18 settings and their S/N ratio values in the third factorial experiment

Trial no.	18 Seven-factor settings (L_{18} orthogonal array)							S/N ratio
	W_d	θ	R_{md}	R_{hl}	R_{out}	R_{in}	\aleph	
1	0.3	55	1.6	0.4	1.3	1.3	Type 1	-25.3152
2	0.3	60	1.6	0.4	1.5	1.5	Type 2	-24.6696
3	0.3	65	1.6	0.4	1.7	1.7	Type 3	-24.1661
4	0.4	55	1.6	0.4	1.5	1.7	Type 3	-24.2937
5	0.4	60	1.6	0.4	1.7	1.3	Type 1	-24.6416
6	0.4	65	1.6	0.4	1.3	1.5	Type 2	-24.3665
7	0.5	55	1.6	0.4	1.7	1.5	Type 3	-23.9263
8	0.5	60	1.6	0.4	1.3	1.7	Type 1	-24.5880
9	0.5	65	1.6	0.4	1.5	1.3	Type 2	-24.2438
10	0.3	55	1.6	0.4	1.5	1.5	Type 1	-24.0759
11	0.3	60	1.6	0.4	1.7	1.7	Type 2	-24.3513
12	0.3	65	1.6	0.4	1.3	1.3	Type 3	-24.2889
13	0.4	55	1.6	0.4	1.3	1.7	Type 2	-24.2672
14	0.4	60	1.6	0.4	1.5	1.3	Type 3	-24.1688
15	0.4	65	1.6	0.4	1.7	1.5	Type 1	-24.6172
16	0.5	55	1.6	0.4	1.7	1.3	Type 2	-24.1554
17	0.5	60	1.6	0.4	1.3	1.5	Type 3	-24.1273
18	0.5	65	1.6	0.4	1.5	1.7	Type 1	-24.5150

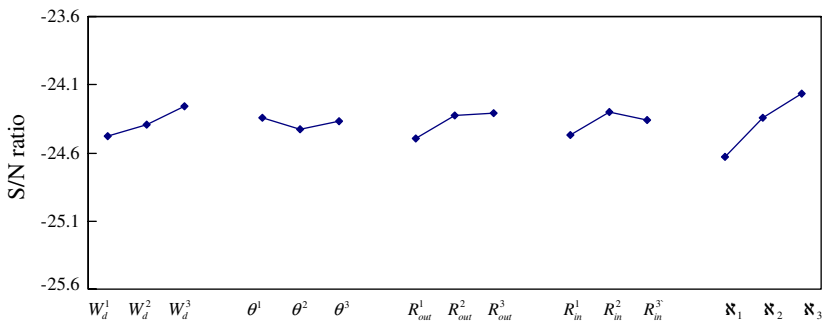


Fig. 8 The line graph of the $AV_{I_{IV}}$ of three levels versus the five respective factors

3.6 The fourth factorial experiment (fractional)

The orthogonal array L_{18} was used again for the four non-fixed factors and the three fixed factors, $R_{hl} = 0.4$, $R_{md} = 1.6$, $\aleph = \text{Type 3}$. The 18 different settings were arranged in the array in Table 7. Similarly, six of the 18 settings appeared in Table 6, so 12 settings were processed by as many ANSYS analyses. Then, the 18 settings' S/N ratio values were determined and presented in Table 7.

The average S/N ratio of the 18 settings, AV_{all} , was calculated as -24.2290 . The line graph shown in Fig. 9 contains four groups of line segments for factors W_d , θ , R_{out} , and

Table 7 The presentation of 18 settings and their S/N ratio values in the fourth factorial experiment

Trial no.	18 Seven-factor settings (L_{18} orthogonal array)							S/N ratio
	W_d	θ	R_{md}	R_{hl}	R_{out}	R_{in}	\aleph	
1	0.3	55	1.6	0.4	1.3	1.3	Type 3	-24.9840
2	0.3	60	1.6	0.4	1.5	1.5	Type 3	-24.4455
3	0.3	65	1.6	0.4	1.7	1.7	Type 3	-24.1661
4	0.4	55	1.6	0.4	1.5	1.7	Type 3	-24.2937
5	0.4	60	1.6	0.4	1.7	1.3	Type 3	-24.0285
6	0.4	65	1.6	0.4	1.3	1.5	Type 3	-24.1983
7	0.5	55	1.6	0.4	1.7	1.5	Type 3	-23.9263
8	0.5	60	1.6	0.4	1.3	1.7	Type 3	-24.1554
9	0.5	65	1.6	0.4	1.5	1.3	Type 3	-24.0307
10	0.3	55	1.6	0.4	1.5	1.5	Type 3	-24.4720
11	0.3	60	1.6	0.4	1.7	1.7	Type 3	-24.1930
12	0.3	65	1.6	0.4	1.3	1.3	Type 3	-24.2889
13	0.4	55	1.6	0.4	1.3	1.7	Type 3	-24.6559
14	0.4	60	1.6	0.4	1.5	1.3	Type 3	-24.1689
15	0.4	65	1.6	0.4	1.7	1.5	Type 3	-24.0159
16	0.5	55	1.6	0.4	1.7	1.3	Type 3	-23.8814
17	0.5	60	1.6	0.4	1.3	1.5	Type 3	-24.1273
18	0.5	65	1.6	0.4	1.5	1.7	Type 3	-24.0895

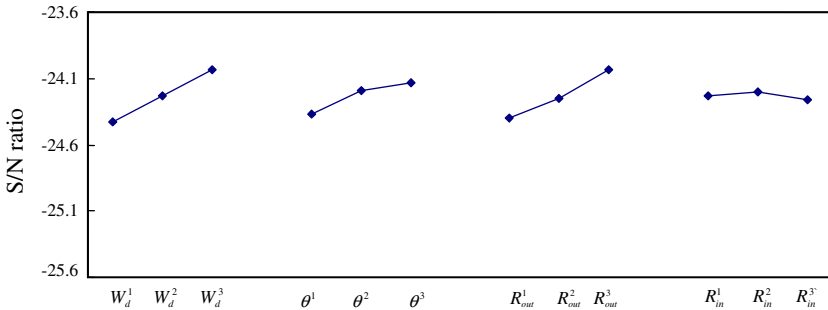


Fig. 9 The line graph of the AV_{lvl} of three levels versus the four respective factors

R_{in} . By taking each factor's largest AV_{lvl} , we determined the best setting in the experiment, composed of $R_{hl} = 0.4$, $R_{md} = 1.6$, $\aleph = \text{Type 3}$, $W_d^3 = 0.5$, $\theta^3 = 65$, $R_{out}^3 = 1.7$, $R_{in}^2 = 1.5$. And the setting's S/N ratio, S/N_{lg} , was determined after completing the ANSYS analysis as -24.4819 .

It was found that S/N_{lg} is less than AV_{all} , which violates the requirement of Step 7(a) of the algorithm. Actually, the Step 7(a) is considered in the proposed algorithm as the fundamental condition required while applying the Taguchi method. Hence, this violation indicates that the orthogonal array, L_{18} , is not suitable any more for screening the dominated one from

the remaining unfixed factors with three levels. Therefore, to process the next design of experiment, we adopted the zoom-in technique that reduces one level in each non-fixed factor as depicted in (b) of the algorithm’s Step 7.

Note the level eliminated from each factor is the one generating the smallest AV_{lvl} , and they are $W_d^3 = 0.3, \theta^1 = 55, R_{out}^1 = 1.3, R_{in}^3 = 1.7$, respectively.

3.7 The fifth factorial experiment (fractional)

After a level was reduced, each of the four factors W_d, θ, R_{out} , and R_{in} retained two levels to be conducted. Therefore, we chose the two-level Taguchi orthogonal array $L_8(2^7)$ for the experiment given three-level factors, $R_{hl} = 0.4, R_{md} = 1.6$ and $\aleph = \text{Type 3}$. Eight settings were formed according to the array’s requirements and displayed in Table 8. After completing the ANSYS analyses for all the settings, we determined their S/N ratios and presented them in Table 8. Meanwhile, the average S/N ratio of the eight settings, AV_{all} , was -24.1783 .

Similarly, a line graph of four segments was shown in Fig. 10, from which the best setting was determined as $W_d^2 = 0.4, \theta^2 = 60, R_{out}^2 = 1.5, R_{in}^1 = 1.3, R_{hl} = 0.4, R_{md} = 1.6, \aleph = \text{Type 3}$. Then, we determined the setting’s S/N ratio, S/N_{lg} , as -24.1688 , which is larger than the AV_{all} .

Table 8 The presentation of eight settings and their S/N ratio values in the fifth fractional factorial experiment

Trial no.	8 Seven-factor settings ($L_8(2^7)$ orthogonal array)							S/N ratio
	W_d	θ	R_{out}	R_{in}	R_{hl}	R_{md}	\aleph	
1	0.4	60	1.5	1.3	0.4	1.6	Type 3	-24.1688
2	0.4	60	1.7	1.5	0.4	1.6	Type 3	-23.8156
3	0.4	65	1.5	1.5	0.4	1.6	Type 3	-24.2860
4	0.4	65	1.7	1.3	0.4	1.6	Type 3	-23.9803
5	0.5	60	1.5	1.5	0.4	1.6	Type 3	-24.1866
6	0.5	60	1.7	1.3	0.4	1.6	Type 3	-24.4699
7	0.5	65	1.5	1.3	0.4	1.6	Type 3	-24.0307
8	0.5	65	1.7	1.5	0.4	1.6	Type 3	-24.4891

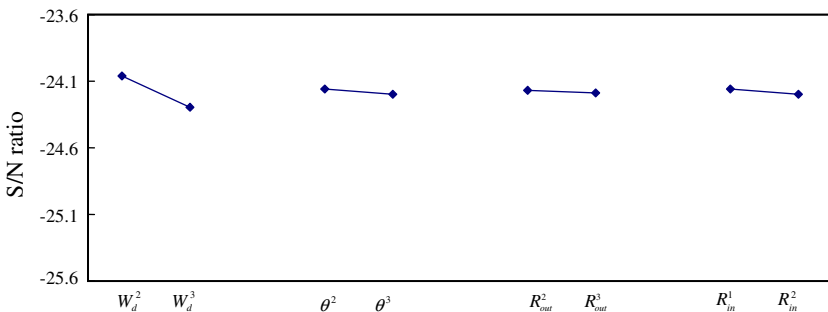


Fig. 10 The line graph of the AV_{lvl} of three levels versus the four respective factors

Accordingly, W_d , having level 3 as the best level, serves as the most influential factor in the experiment.

3.8 The sixth factorial experiment (full)

After the last experiment, there remains three non-fixed factors, θ , R_{out} , and R_{in} . Since the three factors generate at most eight combinations affordable to a full factorial approach. The eight new settings were presented in Table 9, and all the setting's S/N ratios were determined by as many ANSYS analyses and shown in the last column of Table 9; the best setting having S/N_{Ig} of -23.8156 was found in the second row as $W_d^2 = 0.4, \theta^2 = 60, R_{out}^3 = 1.7, R_{in}^2 = 1.5, R_{hl} = 0.4, R_{md} = 1.6, \aleph = \text{Type 3}$.

Table 10 presented the top five largest-S/N ratio settings for the case design conducted by Zeng [11] through 2187-setting trials. Comparing the results between Tables 9 and 10, the best setting determined in the sixth experiment is exactly the overall best one obtained by Zeng.

The variations in both S/N_{Ig} and AV_{all} over the six experiments are shown in Fig. 11. We found that the rate of S/N_{Ig} shows a constant increasing till the value at the last experiment, and the global optimum to this design of hatch corner is obtained. However, the AV_{all} roughly levels off, little affected by a level reduced. The present approach used only 53-setting trials in gaining the overall best setting, shy of 3% the 2187-setting trials by Zeng. Further, we found the best setting by the first experiment, equivalent to the Taguchi approach, finished 366th of 2187 settings.

Table 9 The presentation of eight settings and their S/N ratio values in the sixth factorial experiment

Trial no.	8 Seven-factor settings							S/N ratio
	θ	R_{out}	R_{in}	R_{hl}	R_{md}	\aleph	W_d	
1	60	1.5	1.3	0.4	1.6	Type 3	0.4	-24.1688
2	60	1.7	1.5	0.4	1.6	Type 3	0.4	-23.8156
3	65	1.5	1.5	0.4	1.6	Type 3	0.4	-24.2858
4	65	1.7	1.3	0.4	1.6	Type 3	0.4	-23.9803
5	60	1.5	1.5	0.4	1.6	Type 3	0.4	-24.1903
6	60	1.7	1.3	0.4	1.6	Type 3	0.4	-24.0285
7	65	1.5	1.3	0.4	1.6	Type 3	0.4	-24.2603
8	65	1.7	1.5	0.4	1.6	Type 3	0.4	-24.0159

Table 10 The top five settings of the design by Zeng [11] in a full factorial experiment

	θ	R_{out}	R_{in}	R_{hl}	R_{md}	\aleph	W_d	S/N ratio
Top 1	60	1.7	1.5	0.4	1.6	Type 3	0.4	-23.8156
Top 2	60	1.7	1.7	0.4	1.6	Type 3	0.4	-23.8491
Top 3	55	1.7	1.3	0.4	1.6	Type 3	0.5	-23.8814
Top 4	55	1.7	1.3	0.4	1.6	Type 3	0.4	-23.8958
Top 5	60	1.7	1.3	0.4	1.5	Type 3	0.3	-23.8986

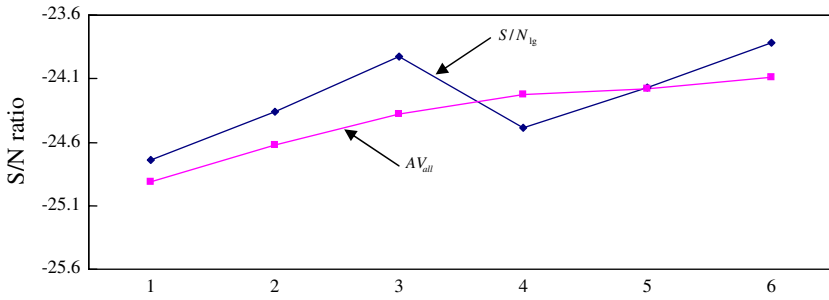


Fig. 11 The variations of S/N_{lg} and AV_{all} versus the factorial experiment

4 Conclusions

By creatively blending several fractional factorial experiments and a full factorial one, we have developed a new, effective and efficient approach for design of experiments. The performance of the proposed approach was verified in designing a hatch corner having a groove with-a-hole in a containership. From the results, we made the following conclusions:

- (1) The proposed approach can find the global optimum of the design of experiment successfully with a much lower percentage of trials to those used in a full factorial approach. For example 3% of the total optional settings were consumed in the design of a hatch corner having a groove with-a-hole.
- (2) Though the Taguchi Technique offers a feasible solution, it hardly provides a global optimum. In the design of a hatch corner having a groove with-a-hole, Taguchi approach's solution ranks 366th of 2187 options. We found the rank of the best setting by the Taguchi approach depends on the problem itself.
- (3) The proposed approach ranks the design parameters by their influences on the objective characteristics of the design. This provides helpful information for designers in deciding to strengthen or reduce the effects of parameters through planning a new design. The benefit was not found in conventional factorial approaches, fractional or full.
- (4) The proposed approach combines the benefits of both factorial and full factorial approaches, effective and efficient, and would serve as a total solution tool for optimization using design of experiments.

The proposed approach has been proven applicable in the design of experiment with a single objective in this study. Further work employing the approach in optimizing multi-objective problems is expected.

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